E 2151 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, AUGUST 2021

Third Semester

Core Course (3)—CALCULUS

(Common for Model I, Model II Mathematics and B.Sc. Computer Applications)

[2013 – 2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. The value of $D^n (ax + b)^n$ is _____.
2. State Maclaurin's theorem.
3. The radius of curvature of $y = e^x$ at the point (0, 1) is _____.
4. How many asymptotes a closed curve has?
5. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ if $f(xy) = (xy - 1)^2$?
6. Find $\frac{dy}{dx}$ at (1, 1) given $x^2 - 2y^2 + xy = 0$.
7. Find the gradient of $f(x, y) = y - x$ at (2, 1).
8. Write the shell formula for finding the volume of the solid generated by revolving the region between and graph of $y = f(x) \geq 0$, $0 \leq a \leq x \leq b$ about y-axis.
9. Define a smooth curve.
10. Write the differential formula for arc length.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*

Each question carries 2 *marks.*

11. Expand $\log (1 + x)$ as a Maclaurin's series.
12. Find the radius of curvature of $y = 3x^2 + 4x$ at (1, 7)?
13. Examine the curve $y = \sin x$ for concavity upwards and concavity downwards and for points of inflexion in $[-2\pi, 2\pi]$.

Turn over

14. Find the asymptotes parallel to the co-ordinate axes of the curve $(x^2 + y^2)\, x - ay^2 = 0$.

15. If $u = \sin(xy)$, show that $x \frac{du}{dx} - y \frac{du}{dx} = 0$.

16. Verify that $\frac{d^2u}{dxdy} = \frac{d^2u}{dydx}$ for $u = x^y + \sin(xy)$.

17. Verify Euler's theorem for $u = x^3 - y^3 + 3x^2y$.

18. Evaluate $\int_{-1}^{1} 3x^2 \sqrt{x^3 + 1}\, dx$.

19. Find the area of the region enclosed by $y = x^2 - 2$ and $y = 2$.

20. The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ is revolved about the y-axis to generate a solid. Find the volume of the solid?

21. Evaluate $\int_1^{\log 8} \int_0^{\log y} e^{x+y}\, dxdy$.

22. Find the area enclosed by $r^2 = 4 \cos 2\theta$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Show that if $y = \sin(m \sin^{-1} x)$, $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0$.

24. For the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$. Prove that the radius of curvature $\rho = 4a \cos(\frac{t}{2})$.

25. Find the envelope of the straight lines $\frac{x}{\sin\alpha} + \frac{y}{\cos\alpha} = l$.

26. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \log s$, $z = 2r$.

27. Find $\frac{\partial w}{\partial x}$ if $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$.

28. Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.

29. Find the area of the region enclosed by $y = 2 \sin x$ and $y = \sin 2x$, $0 \le x \le \pi$.

30. Evaluate $\iint_R e^{x^2 + y^2}\, dydx$.

31. Find the average value of F $(x, y, z) = x^2 + 9$ over the cube in the first octant bounded by the co-ordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find $y_n(0)$ when $y = \log (x + \sqrt{1+x^2})$.

(b) Find the co-ordinates of the centre of curvature at a point (x, y) of the Parabola $y^2 = 4ax$.

33. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3 and requiring the least amount of material for its construction.

34. (a) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis and the second plane crosses the first plane at a 45° angle at the centre of the cylinder. Find the volume of the wedge?

(b) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about the x-axis.

35. (a) Evaluate $\int_0^{2\pi}\int_0^{1}\int_r^{1/\sqrt{2-r^2}} 3d2rdrd\theta$.

(b) Evaluate $\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

$(2 \times 15 = 30)$